## 2024 Canadian Economics Olympiad - National Finale

Canadian Economics Olympiad is the only representative of the International Economics Olympiad in Canada authorized to select Team Canada for participation in the 2024 International Economics Olympiad. The International Economics Olympiad is an annual competition in Economics organized for high school students. It is intended to stimulate the activities of students interested in economics, business, and finance by way of creative problem-solving.

## Please read the following information carefully and follow the submission instructions.

The 2024 Canadian Economics Olympiad consists of two rounds. This is Round 2 - National Finale. This round consists of 7 long questions. You must provide detailed solutions to all questions unless specified otherwise: answers without solutions will not receive any credit. Some of the questions (as specified in the questions) require that you type your solution. For the remaining questions, you can either hand-write or type your solution.

You must submit your solutions as a single PDF file by email to canadian.economics.olympiad@gmail.com. If you need to change your submission before the deadline simply email a new submission. Only the latest submission will be graded. You may find www.ilovepdf.com to be helpful in transferring almost any file format to PDF, merging several PDF files into one, and reducing the size of PDF files. The latter may be especially helpful to reduce your file size to under 5MB so that it can be sent as an email attachment (not as a link to a Google Drive). You may also find some free PDF scanning apps for your phone (highly recommended!)

IMPORTANT: Please, make sure you submit a single PDF file. If you type all of your solutions in MS Word - simply save the file as PDF. If you type some of your solutions and hand-write others, first make a PDF file of your typed solution, then scan your hand-written solutions into a PDF file (or take a photo of it and convert it to PDF either using www.ilovepdf.com or any other means) and then merge all PDF files into a single file (either using www.ilovepdf.com or any other means)

IMPORTANT: make sure your PDF file is not too large, preferably under 5MB so that it can be sent by email. To do so, scan your hand-written solutions in black-and-white mode with medium or low resolution, or, if you take photos, use low resolutions. Then, if your PDF file ends up being too large, use www.ilovepdf.com or any other means to compress it.

VERY IMPORTANT: If you use Apple products, make sure your PDF file can be opened on PC. To be safe, use www.ilovepdf.com

The test will be emailed to you on April 27 at or around 5 pm (Toronto time) and you must submit your file by email by 8 pm (Toronto time). The 3 hours that you have to solve the test include the time needed for you to prepare and send the PDF file, so, make sure you (1) are familiar with how to prepare a single PDF file and compress it if needed (2) practice making such file using 2 pages of types and 6 pages of handwritten text to make sure you can produce a PDF file of a size under 5MP, and (3) leave sufficient time at the end of the test to complete your submission in time. Submissions received between 8 pm and $8: 15 \mathrm{pm}$ will receive a progressive penalty. Submissions received after $8: 15 \mathrm{pm}$ will not be accepted.

You are allowed and encouraged to use any online and offline resources during the contest, but you are not allowed to seek help from other people or Artificial Intelligence (e.g. chat GPT). You may not ask questions (either directly or indirectly) related to this contest or any economics, finance, or accounting topics before you submit your test.

If you have questions about any part of the exam during the contest, please email canadian.economics.olympiad@gmail.com. In case any major mistakes are found, an email will be sent to all contestants, so, make sure you check your emails frequently. To prevent your questions and submissions from being lost in the email chain, DO NOT REPLY to our emails. Instead, always send a new email. For submission, put "Submission" in the subject of the email, and for questions, put "Questions" in the subject line.

## Problem 1 ( 25 points)

Consider 2 firms producing identical products in a market with a demand function $P(Q)=$ $300-Q$. The firms compete by simultaneously choosing their production level. The production costs are zero
a) (4 points) Prove that in equilibrium each firm earns $\$ 10,000$ profit.
b) (4 points) Prove that if firms collude and form a cartel, each firm can earn $\$ 11,250$
c) ( 5 points) Assume the firms play this game for 100 periods (e.g. each period is a year and each year firms choose their current production level for the year). At the beginning of each period, firms choose their production level for that period and can base their decision on the history of production (both its own and the other firm's) up to that date. Assume also there is no binding agreement that the firm can enter into between themselves. What can you say about the total profit of each firm in equilibrium?
d) (6 points) Now assume firms play this game forever but future profits are discounted back with a very small annual interest rate $r$. As before, at the beginning of each period, firms choose their production level for that period and can base their decision on the history of production (both its own and the other firm's) up to that date, and assume there is no binding agreement that the firms can enter into between themselves. Can both firms receive $\$ 11,250$ profit each period in equilibrium? If yes, explain why and be very detailed about the equilibrium strategies. If no, explain why.
e) (6 points) Using the same information as in part (d), can one of the firms receive $\$ 20,000$ profit each period in equilibrium? If yes, explain why and be very detailed about the equilibrium strategies. If no, explain why.

## Problem 2 ( 15 points)

Consider a competitive market with a standard U-shaped cost structure for each firm so that the Average Variable Costs (AVC) and Average Total Costs (ATC) curves intersect the Marginal Cost (MC) curve in their respective minimums. Assume the market demand curve is downwardsloping. Currently, there are a large number N of identical firms operating on the market
a) Now, assume $\mathrm{N}_{1}$ firms (we call them "innovative" firms) were able to change their operation model to reduce their fixed costs, while the remaining $\mathrm{N}_{2}=\mathrm{N}-\mathrm{N}_{1}$ firms (we call them "traditional" firms) still operate with the original high fixed costs. Assume $\mathrm{N}_{1}$ is sufficiently small so that in the new long-run equilibrium both types of firms still operate in the market, but $\mathrm{N}_{1}$ is not negligible.
a1) (2 points) Will the production by a single "traditional" firm still operating in a new long-run equilibrium be higher than, lower than, or the same as in the original equilibrium? No explanation is needed, only the answer
a2) (2 points) Will the production by a single "innovative" firm in a new long-run equilibrium be higher than, lower than, or the same as in the original equilibrium? No explanation is needed, only the answer.
b) Assume the innovation in part (a) was not the reduction in fixed costs but a reduction in marginal costs.
b1) (2 points) Will the production by a single "traditional" firm still operating in a new long-run equilibrium be higher than, lower than, or the same as in the original equilibrium? No explanation is needed, only the answer.
b2) (2 points) Will the production by a single "innovative" firm in a new long-run equilibrium be higher than, lower than, or the same as in the original equilibrium? No explanation is needed, only the answer.
c) (7 points) Now, continue to assume that the reduction was in marginal costs, but assume that $\mathrm{N}_{1}$ was not too small. In particular, assume in the new long-run equilibrium exactly $\mathrm{N}_{1}$ of innovative firms are operating in the market and all of the traditional firms decided to leave the market. Let $P_{0}$ and $P_{1}$ be the market price in old (before the innovation) and new (after the innovation) long-run equilibria and let C be the minimum ATC of the innovative firms. What can you say about the relationship between $\mathrm{P}_{0}, \mathrm{P}_{1}$, and C ? Be as specific as possible. No explanation is needed, only the answer.

## Problem 3 (15 points):

A firm is expected to pay $\$ 5$ dividend next year, no dividends in years 2-9, \$40 dividend in year 10 , no dividends in years $11-19, \$ 30$ in year 20, and then dividends will continue to grow at a constant rate of $\mathrm{g} \%$ per year forever. The firm's current stock price is $\$ 100$, its systematic risk is $\beta=1.4$, the risk-free interest rate is $6 \%$, and the expected market return is $11 \%$
a) (9 points) Find the growth rate $g$
b) A capital gain yield is defined as a percentage increase in price during the year.
b1) (2 points) Find the capital gain yield during the seventh year.
b2) (2 points) Find the capital gain yield during the $27^{\text {th }}$ year.
b3) (2 points) Find the capital gain yield during the first year.

Problem 4 (5 points): A utility of a person is given by $u(C, L)=C^{\alpha} L^{1-\alpha}$, where C is consumption and L is leisure (where leisure is equal to the amount of time the person does not work). If the hourly wage is doubled, how C and L will change?

## Problem 5 (15 points):

Consider a population where each citizen lives for two periods: one period where the citizen works and saves money, and the other period where the citizen is in retirement. While working, the citizen is paid a wage of $\$ 100$ and saves a part of their wage for their retirement. Assume saving earns interest of $10 \%$ and the total utility of a person is equal to $u\left(c_{1}\right)+\frac{u\left(c_{2}\right)}{1.1}$, where $c_{1}$ and $c_{2}$ are consumption levels during working and retirement periods and $u(c)$ is increasing utility function with decreasing marginal utility. The initial population in period 1 is 100 , and the population grows by $8 \%$ each period.
a) (2 points) What is the ratio of $\frac{c_{1}}{c_{2}}$ ?
b) (5 points) Regardless of your answer in part (a), assume $c_{1}=c_{2}$. How much money did each person save when they work?
c) (5 points) Now, assume that the government introduces a pension plan, where all retired citizens receive a pension equal to half of their wage while they are working. The pension is funded by imposing a tax on all working citizens. If the tax burden on all citizens is equal, what percentage of workers' wages are taxed in the new long-run equilibrium?
d) (3 points) In the new long-run equilibrium, will people continue to save a portion of their after-tax income while they work or spend it all on consumption and rely only on the pension when retired? Explain why.

## Problem 6 (15 points):

The government is concerned that, despite having a competitive insurance market for homeowner insurance, there are many insurance policies with high deductibles. Thus, it passed the law requiring all homeowner insurance policies to have zero deductibles. How this law will affect (will make better off, will make worth off, or will not affect the well-being) different parties affected by this law? Assume that people's behavior about how they care for their houses does not change. Please, be detailed but avoid lengthy writing. Bullet-point answers are preferred, explanation is required. You must type your answers (no handwritten answer to this question!)

## Problem 7 (10 points):

In old movies, you can see how business owners want to create an image of success by driving expensive cars and wearing expensive watches. Assume, indeed, that the probability of getting new clients positively depends on the public perception of success. Assume, however, that even an unsuccessful business owner can find the money to buy an expensive car and watch, even if it costs a higher portion of his wealth. So, why he does not do it? I.e., which standard economics assumption makes him unwilling to mimic the behavior of successful businessmen in order to increase the chance of getting new clients (assume the benefit of new clients is the same for successful and unsuccessful business owners). Please, be detailed but avoid lengthy writing. A 13 sentence answer would be sufficient, but you may write a bit more if it helps to prove your point. You must type your answers (no handwritten answer to this question!)

